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THE DECEASED IS IN GOOD HEALTH [1]

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"The Deceased Is In Good Health" is the name of funny black comedy-mystery written by a contemporary poet and playwright, Gaetano di Maio, born in Naples on 18 August 1927.

I remembered this comedy, when I saw for the first time on the internet, in the www.demo.istat.it site, "Mortality Tables of the Italian Population by Province and Regions of Residence – for the year 1998" that were published by ISTAT (the National Statistics Institute-Roma) in the series: Information n. 19 – 2002. In the notes on methods, ISTAT gives particular importance to some innovations with respect to the tables of mortality elaborated in the past. Among other things, these innovations mainly regard: a) the unification of the methodology of calculation between the national tables and the regional tables; b) the presentation in a single volume of the national, regional and provincial tables; c) the use of a new model for the estimate of the probability of death during senile years.

In this note we intend to examine some of the details and curious aspects of the ISTAT tables which characterize their construction, particularly in the senile ages.

As a premise, nevertheless we want to report some reflections that were made at different times in Italy about the opportunity, or the lack of the same, to assume a given age that would be the extreme life span.

For example, in the "Table of mortality of the Italian population" (Istat-Serie VI - vol. VIII, for the year 1931) Gini and Galvani observed that, in general, the quotients of mortality for the senile age in general could not be determined in a correct manner given the uncertainty that the relative data offer, both in the number of deaths and the number of people living at that age.

The two authors thus ask if it would be best to not calculate this quotient, setting, or example, an age limit of life, indicated with ω, for which ${}_{\omega-x}q_x = 1$, or to determine the senile quotients with the extrapolation that: "…conform to certain more or less plausible hypotheses", and that more or less reflect the arbitrariness of the hypotheses that they are based upon. [2]

In the same study in 1931, Gini and Galvani observed, furthermore, that in the tables of some European countries, values of the limit for the duration of human life had been set, deduced from research carried out in the classes of extreme age.. They also recognized that in any method followed for their calculation, these percentages would have a rather limited importance. However, precisely due to the weaknesses offered by the statistical survey of the data, they believed that : "…the forced suppression of survivors at a deductive established age seems truly excessive.", and

that: "…it is, in fact, unnecessary that the probability of death should read value 1, because the l_x assume, from a certain age on, practically zero values".

Their tables: 1881-1882, 1899-1902, 1910-1912, and 1921-1922, go up to an age including 100-108 years, and they end just as the number of survivors descends to a value that is less than one.

In the "Table of mortality of the Italian population 1950-53 and 1954-57", ISTAT 1959, again regarding the probability of death in senile age, ISTAT observes: "Many statisticians set a precise limit of the duration of the human life, generally calculated from observation. Because a generation is extinguished with practical certainty, it is not, however, necessary that the probability of death should reach the precise value of the unit that corresponds to a certain age; the tables of mortality cannot suppose, that is, to say at what age it would be necessary that the last survivor dies, but only in what way do you come near to a practical certainty of the probability that the last survivor will not reach a certain age". In substance, they repeat the same concepts from the Gini and Galvani tables.

The ISTAT tables for 1960-62 also assume that the probability of death grows, tending asymptotically to the unit, and they repeat in note 2) page 19, the same phrase indicated above (Istat, 1959), or rather: "Many statisticians set a precise limit… … .. reaching a certain age". In the ISTAT tables for 1960-62 they generally arrive up to an age of 109 years, where the table ends, when the number of survivors is included between 0 and 1. The 1977-1979 tables they end with an age of 104 years with a whole number of survivors.

ISTAT tables 1979-83: here they follow an interpolation for 4 points of the function of survival of Gompertz-Makeham:

$$l_x = ab^x c^{y^X}$$

in ages 65, 75, 85 and 95. Here, the tables end at 105 years of age with a whole number greater of zero.

In the 1992 ISTAT tables, also elaborated with Gompertz-Makeham, the biometric functions are presented up until the age of 109 years.

In the mortality tables of 1998 that we are examining, ISTAT assigned its calculations according to the following outline: "Although after the age of 95 years incidences of death are rare, if not actually unique, as we slowly come near to the thresholds of extreme survival, it is necessary to be able to use a demographic model that should be aimed at describing the regular trend of the probability of death. The use of a model is, in this case, aimed at closing the tables of mortality at the age of 125 years."

But the real news, according to ISTAT, is in the model chosen for representing the evolution of the probability of death, which is the one by Kannisto (1998), presented in the formula:

$$q_x = \alpha e^{\beta x} / \left(1 + \alpha e^{\beta x}\right) \qquad \textbf{[1]}$$

and defined as the function belonging to the family of logistic curves.

In [1], although it is not specifically stated, the value of the probability of death in age x is assimilated to the value of the instantaneous rate of mortality at the age of x, to which Kannisto refers in his study, setting, on various occasions:

$$\mu_x = c + \frac{ae^{bx}}{1+ae^{bx}} \quad \textbf{and even more simply:} \quad \mu_x = \frac{ae^{bx}}{1+ae^{bx}} \qquad [2]$$

were μ_x indicates the force of mortality or the instantaneous death rate at age x.

We observe, at this point, that the "Kannisto model" is none other than the traditional logistic function in the formula:

$$q_x = 1/\left(1+\frac{1}{a}e^{-bx}\right) \qquad [3]$$

Since in [3] the higher asymptote is set with the value of one, for the adaptation of the function of the data observed, the value of the two parameters of a and b remain to be set. In order to verify this, if it should be necessary, it is enough to divide the numerator and denominator by ae^{bx}.

For the estimate of a and of b, ISTAT linearized the [3] , for which, maintaining the notations used, obtains:

$$\ln\left(q_x / p_x\right) = \ln(a) + bx \qquad [4]$$

noting that the [4] could also be written as:

$$\operatorname{log\,it}\ q_x = \ln(a) + bx \qquad [5]$$

as the term coined by Berkson (1944).

From the standard probability of dying, calculated on the national basis for males and females, the procedure continues on to the regional probability through the equivalence;

$$q_x^r = q_x^s \cdot K_x \qquad \textbf{where} \qquad K_x = \left(q_{92}^r / q_{92}^s\right)^{(125-x)/33} \qquad [6]$$

is an exponential function of the relationship between the regional probability at 92 years and the standard one at the same age, which is developed in the following years on the basis of the fraction of time of the interval of 33 years (between 92 and 125). At the age of 92 years the value of K is q_{92}^r / q_{92}^s for which there are no variations in regional probability; at 125 years, K is equal to 1 and, therefore, the two probabilities of death have the same value, not for this are they equal to 1.

Again from the tables of ISTAT (Italy, 2002), one observes that the probability of death for the two sexes reaches values of 877.7 per 1000 in males and 824.2 in females at the age of 119 years (Figure 1).

For ages of more than 125 years, the probability of death, both national and regional, continue their climb, nearer and nearer to 1, with values of 0.999 per 1000 between 150 and 160 years of age, to which one cannot attribute any significance beyond the numerical.

Besides the doubtfulness of the process that for the regional tables considers a single value q_{92}^r for the connection with the national standard at the same age, the

strangest thing, that which has led us to study these tables, is the fact that in them you arrive at a certain very advanced age in which there are zero survivors,. which continue to manifest probability of death, of survival and values of average life also in the following ages.

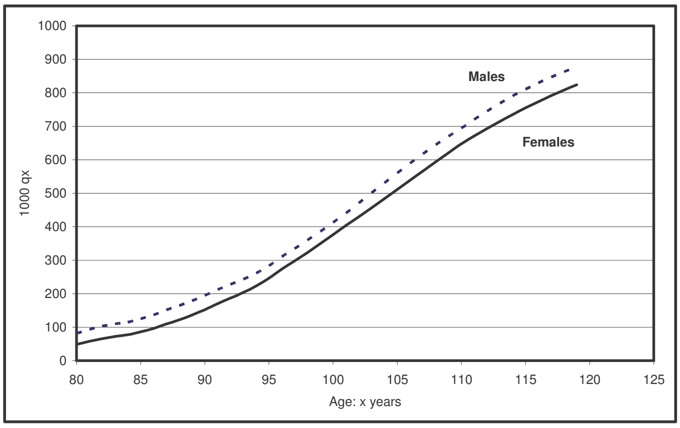

Figure 1. Probability at age x of dying before reaching age x+1. Life tables ISTAT, 1998. Italy, female population.

For example, for the national life table for females, at the beginning of 112 years you have zero survivors and zero deaths, but the probability to survive up to 113 years is 0.3066, and the expectation of life of almost a year (being e_{112}=0.92).

This is also repeated in the following ages (Table 1).

Table 1 Life table ISTAT 1998. Italy, female population, with l_o = 100.000 and world births per year l_o = 131.993.000

Age x	l_x	d_x	1000 q_x	Lx	e_x	l_x	d_x
0	100000	531	5.307	99501	81.76	131993000	604961
.	. .	. .	. . .	. .	. . .	. . . .	. .
105	96	49	511.296	71	1.37	126523	64690
106	47	25	538.861	34	1.29	61833	33319
107	22	12	566.433	15	1.21	28514	16151
108	9	6	593.888	7	1.14	12363	7342
109	4	2	621.108	3	1.08	5021	3118
110	1	1	647.892	1	1.02	1903	1233
111	1	0	671.051	0	0.97	670	450
112	0	0	693.358	0	0.92	220	152
113	0	0	714.835	0	0.88	68	49
114	0	0	735.429	0	0.85	19	14
115	0	0	755.102	0	0.82	5	4
116	0	0	773.826	0	0.78	1	1
117	0	0	791.586	0	0.76	0	0
118	0	0	808.379	0	0.73	0	0
119	0	0	824.212	0	0.71	0	0

One should say that the "hidden" decimals need to be considered. But we have also tried to apply these probabilities to world births per year in the period of 1995-2000 (Data UN-2001) which result in the number 131,993,000. From the second part of Table 1 one observes that also beginning from a notable number of entries in the process of mortality, one always reaches the absence of survivors that indicates the end of this process.

In substance, it seems to us that extending the data of the table of mortality in age in which the number of survivors is zero should be avoided in the presentation of the same tables, also because, among other things, it is completely useless.

Gaetano di Maio, in his comedy, said that: "the dead man is in good health". ISTAT, however, informs us that if of a community of individuals, subjected to a natural process of elimination, there are zero remaining at one of the senile ages, the cadavers can also hope to live for a while, or rather that "they are almost in good health".

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(1) Free translation of the original name: "Il morto sta bene in salute" of the comedy written by Gaetano di Maio.

(2) Symbols used in the text: d_x = deaths between age x and x+1;; e_x = expectation of life at age x; l_x = number of survivors at age x out on original cohort of 100,000 births; q_x = probability at age x of dying before reaching age x+1.

(3) For more comparisons on this model, ISTAT suggests, on page. 16 of its volume (2002), to consult the methodological index of the same volume which, however, seems to us not to exist.

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