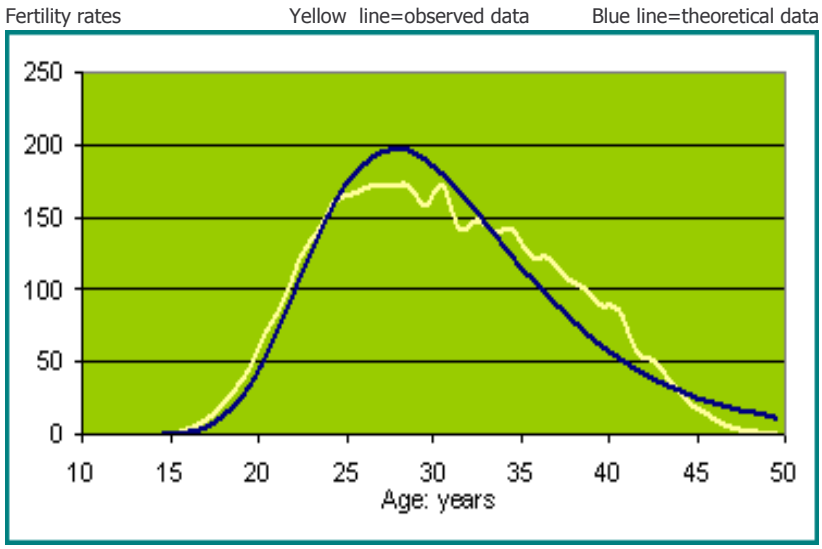
FERTILITY FUNCTIONS

(Source:Petrioli Luciano,"*PRODEMOG 3.0-Demographic software for Windows*", EMMECI-SIENA-ITALY,(2000).

G U M B E L



Age-specific fertility rates: Italy, year 1934

The Gumbel distribution is:

$$F(x)= C \cdot e^{-e^{-[(X-N)/M]}} \qquad [1]$$

While the density function is:

$$f(x) = (C/M) \cdot e^{-[(x-N)/M]} \cdot e^{-e^{-[(X-N)/M]}} \qquad [2]$$

In the Gumbel function, the mean and variance are:

$$MED=N+M \cdot 0.577215 \; ; \; DS = (M^2 \cdot \pi^2)/6 \qquad [3]$$

From which the values of the parameters M and N are:

$$M= \frac{\sqrt{6 \cdot DS}}{\pi} \quad ; N=MED-M \cdot 0.577215 \qquad [4]$$

Regarding the functions of Gompertz and Gumbel, let us observe that, although formally they are different, in substance, they arrive at the same result (*) as we can demonstrate, indicating the two functions in the following way:

Gompertz $F(x) = C \cdot A^{B^{X-TV}}$ Gumbel F(x)= $C \cdot e^{-e^{-[(x-N)/M]}}$ [5]

Indeed, we see that by substituting in the function of Gompertz the relations:

B= $e^{-(1/M)}$; A= $e^{-e^{(N-TV)/M}}$ [6]

We obtain the function of Gumbel in the form indicated above. This permits the extraction of the parameters of Gompertz, by knowing the value of the parameters of Gumbel and vice versa.

The parameters M and N can be, hence, determined by means of the relations:

M= $-1/(\ln B)$; N= $-(\ln(-\ln A))/(\ln B) + TV$ [7]

This is useful in determining the parameters of Gumbel when you know the parameters of Gompertz.